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Mixed FE Analysis of Viscoelastic Cylindrical Helixes

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Abstract. In this study, analysis of viscoelastic cylindrical helixes with circular and square cross section is investigated by using the mixed FEM based on Timoshenko beam theory. The Kelvin model is used for the viscoelastic behavior. The analysis is performed in the Laplace domain and the results are transformed back to time domain numerically by Modified Durbin algorithm. The outcome is quite satisfactory besides the necessary engineering precision.

Keywords: Viscoelasticity, Mixed finite element method, Laplace, Cylindrical helix
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INTRODUCTION

The materials, such as plastics, natural and synthetic fibers, metals at elevated temperatures, show viscoelastic behavior. In the literature, one of the best known mechanical models for representing viscoelastic material behavior is the Kelvin model. The theoretical foundations for viscoelasticity are well established by many researches, among them, Fung 1965, Malvern 1969, Flügge 1975 and Christensen 1982 can be referred. The Laplace transform approach (Chen 1995, Aköz and Kadıoğlu 1999, Temel *et al*. 2004, Argeso *et al*. 2011a, Eratlı *et al*. 2011) have been used in the solution of the various viscoelastic structural elements. In this study, a mixed finite element formulation based on the Timoshenko beam theory is used for dynamic analysis of viscoelastic cylindrical helixes with circular and square cross section. The solutions are carried out in Laplace transform space (LTS). Viscoelastic material behavior is simulated by Kelvin model (see Fig. 1) using correspondence principle. The results are transformed back to time domain numerically by Modified Durbin algorithm (Dubner and Abate 1968, Durbin 1974).

FIELD EQUATIONS AND FUNCTIONAL IN LAPLACE TRANSFORM SPACE

The viscoelastic properties of a body may be accounted for through the use of the correspondence principle (Fung 1965, Malvern 1969). For an isotropic viscoelastic material the Poisson's ratio $\bar{\upsilon}$ and $\bar{E}$ are transformed material constants in Laplace domain. Explicit form of the viscoelastic material properties exists in Eratlı *et al*. (2011). The field equations based on Timoshenko beam theory and the details about the helix geometry can be found in Omurtag 1990, Omurtag and Aköz 1992 and

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Olgun 2004. Variables of the two node element at Frenet coordinates are; displacement vector $\bar{\mathbf{u}} = u_t\,\mathbf{t} + u_n\,\mathbf{n} + u_b\,\mathbf{b}$, the rotational vector $\bar{\boldsymbol{\Omega}}$, force vector $\bar{\mathbf{T}}$ and moment vector $\bar{\mathbf{M}}$, respect to the centroidal axes of the cross section, $\bar{\mathbf{T}}$ and $\bar{\mathbf{M}}$ are the force and the moment vectors in the Laplace space, ρ is the density of material, A is area of the cross section, $\mathbf{I}$ is the moment of inertia, $\bar{\boldsymbol{\gamma}}$ is the unit shear vector, $\bar{\boldsymbol{\kappa}}$ is the unit rotation vector, $\bar{\mathbf{C}}_\gamma$ and $\bar{\mathbf{C}}_\kappa$ are creep compliance matrices. $\bar{\mathbf{q}}$ and $\bar{\mathbf{m}}$ are the distributed external force and moment vectors (Eratlı *et al.* 2011). Hence the corresponding functional in the Laplace domain yields to:

$$\left.\begin{aligned}\mathbf{I}(\bar{\mathbf{y}}) = &-[\bar{\mathbf{u}},\bar{\mathbf{T}}_{,s}] + [\mathbf{t}\times\bar{\boldsymbol{\Omega}},\bar{\mathbf{T}}] - [\bar{\mathbf{M}}_{,s},\bar{\boldsymbol{\Omega}}] - \tfrac{1}{2}[\bar{\mathbf{C}}_\kappa\bar{\mathbf{M}},\bar{\mathbf{M}}] - \tfrac{1}{2}[\bar{\mathbf{C}}_\gamma\bar{\mathbf{T}},\bar{\mathbf{T}}] + \tfrac{1}{2}\rho A z^2[\bar{\mathbf{u}},\bar{\mathbf{u}}]\\ &+\tfrac{1}{2}\rho z^2\left[\mathbf{I}\bar{\boldsymbol{\Omega}},\bar{\boldsymbol{\Omega}}\right] - [\bar{\mathbf{q}},\bar{\mathbf{u}}] - [\bar{\mathbf{m}},\bar{\boldsymbol{\Omega}}] + [(\bar{\mathbf{T}}-\hat{\bar{\mathbf{T}}}),\bar{\mathbf{u}}]_\sigma + [(\bar{\mathbf{M}}-\hat{\bar{\mathbf{M}}}),\bar{\boldsymbol{\Omega}}]_\sigma + [\hat{\bar{\mathbf{u}}},\bar{\mathbf{T}}]_\varepsilon + [\hat{\bar{\boldsymbol{\Omega}}},\bar{\mathbf{M}}]_\varepsilon\end{aligned}\right\}\quad (1)$$

FINITE ELEMENT FORMULATION

In finite element formulation as stated by Omurtag 1990 and Omurtag and Aköz 1992, linear shape functions ($\phi_i = (\theta_j - \theta)/\Delta\theta$, $\phi_j = (\theta - \theta_i)/\Delta\theta$) are used, where the subscripts represent node numbers of the beam element, $\theta_j > \theta_i$ and $\Delta\theta = (\theta_j - \theta_i)$. This element has two nodes with 2×12 degrees of freedom. $\{u_t, u_n, u_b, \Omega_t, \Omega_n, \Omega_b, T_t, T_n, T_b, M_t, M_n, M_b\}$ is the nodal unknowns of the mixed finite element.

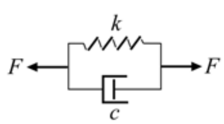

FIGURE 1. Kelvin's model

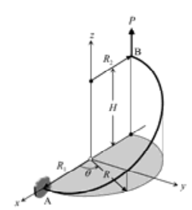

FIGURE 2. Cantilever helical beam

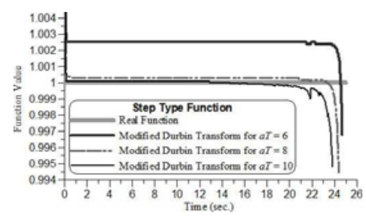

FIGURE 3. Numerical convergence of transformation

NUMERICAL EXAMPLES

Example 1 : Convergence of the Numerical Transformation Algorithm

The inverse Laplace transformation is performed numerically by means of modified Durbin's algorithm. In order to verify the numerical parameter $5 \le aT \le 10$ of the numerical algorithm, a numerical convergence test is performed. For this purpose, a known Laplace transformed functions is inverted to the time space numerically for various values of *aT* and these results are compared by the exact results in order to attain a final decision for the rest of this study. The solutions are performed for the step type loading and the parameters *aT* = 6, 8 and 10 are used in the time interval $0 \le t \le 25\text{second}$. As a final conclusion about the parameters of the Durbin's modified inverse Laplace transformation, the error of the result due to the numerical parameter *aT*=6 is negligible compared to *aT*=8 and 10 but the consumed computation time increases considerably when aT is increased from 6 to 10. Also the numerical behavior of the solution based on aT=6 is more stable. Hence in the rest of this study aT=6 is used (see Figure 3).

Example 2 : Viscoelastic Helicoidal bar

In the dynamic analysis of viscoelastic helixes, a cylindrical helix with circular and square cross-sections is considered (see Figure 2). The viscoelastic material behavior is simulated by using Kelvin's model (see Figure 1). Material and geometrical properties are: the modulus of elasticity $E = 2.06\times10^{11}\ \text{N/m}^2$, Poisson ratio $\upsilon = 0.3$, the number of active turns $n = 0.5$, the height of the helix is H=300 cm, the radius of the helix R_1=R_2=200 cm, the dimension of square cross-section of helix $a = 12$ cm and the diameter of the circular cross-section $D = 13.54$ cm. The net areas of the square and circular cross-sections are kept constant. The density of material ρ=7850 kg/m^3, the damping ratio $f = 0.02$, the intensity of unit step load P=10^6 N. Figure 4 - Figure 7 show time variation u_z at point B and T_z, M_y, M_z at point A,respectively. The maximum peak of vertical displacement at the tip of the helicoidal spring is measured at t=0.23 sec. In the comparison of the displacement u_z of viscoleastic helix with circular and square cross-sections for t=0.23 sec., it is seen that the displacement of the helix with the square cross section is increased by 6% with respect to the circular cross-section. On the other hand, stress resultants are not affected from the change of the cross-section (from circular to square). Results of the square section was verified by other researches existing in the open literature in Eratlı *et al*. 2011.

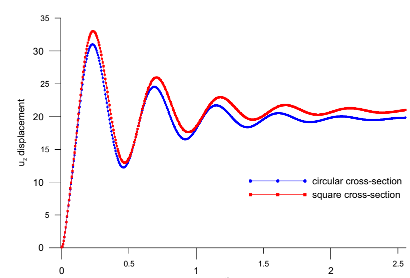

FIGURE 4. The variation of u_z displacement

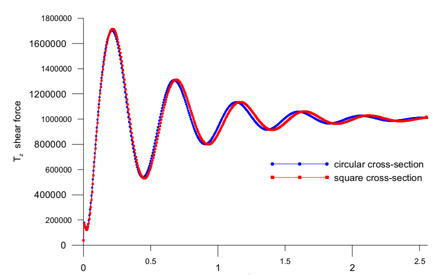

FIGURE 5. The variation of T_z shear force

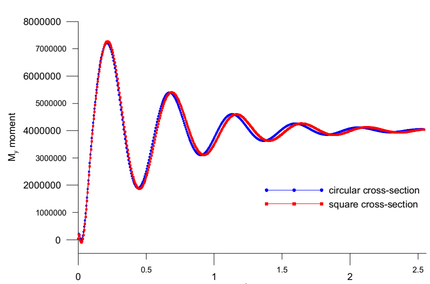

FIGURE 6. The variation of M_y moment

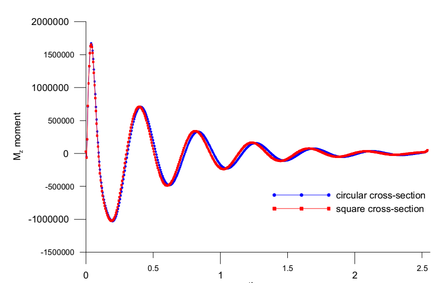

FIGURE 7. The variation of M_z moment

CONCLUSION

A mixed finite element formulation is developed for the dynamic analysis of viscoelastic cylindrical helical beams based on Timoshenko beam theory. The formulation is accomplished in LTS and the viscoelastic properties of a body are accounted using the correspondence principle. The viscoelastic material behavior is simulated by using Kelvin's models. In this study, an example problem is considered with a constant cross-sectional area (circular and square cases are employed) is analyzed with mixed FEM in LTS. The results obtained in transform space are inverted back to time space using Modified Durbin's algorithm and results of these two cross-sections are compared. The obtained results are quite reasonable compared to the existing studies.

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