Flexure of the Moderately Thick Elliptic Plates on Arbitrarily Orthotropic Elastic Foundation

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Abstract

The presented study purposes to deal with bending of moderately thick elliptic plates resting on arbitrarily orthotropic elastic foundation. Plate-foundation interaction field equations are based on the Mindlin plate theory and Pasternak elastic foundation model. A mixed finite element formulation is adopted using Gâteaux differential. A four nodded isoparametric C0 class element is used with eight unknowns as, one deflection, two rotations, two transverse shear forces, two bending moments and a torque. At the first, an effort is made to verify the formulation by comparing results with existing literature and also a convergence for the numerical analysis is presented. Parametric studies are dealt with bending of clamped elliptic Mindlin plates under uniform load and resting on arbitrarily orthotropic Pasternak foundation. Observations of the bending responses of the plate are made for various ellipticity parameters and for various configurations of orthotropic foundation.

Keywords: *mixed finite element, Mindlin plate, bending, Pasternak foundation, elliptic plate.*

1 Introduction

Design procedure of plates requires good understanding of mechanical behavior of plates under operating conditions. Many researchers studied bending, vibration and buckling behavior of plates in many different aspects, especially for rectangular and circular plates. Analysis of bending behavior of elliptical plates have also attracted interests of many researchers. Parnes (Parnes 1988) analyzed clamped and moderately elliptic thin plates subjected to eccentric loads by presenting a higher-order boundary perturbation method (BPM) to derive the Green's function by means of an eccentric source in an elliptic domain. Very recently Altekin (Altekin 2010) optimized bending of orthotropic super-elliptical plates to minimize the maximum deflection of plates by positioning of point supports locations in plates domain.

One of the most commonly faced operating conditions for plates is the interaction with a foundation. Because a foundation interaction degenerates the mechanical behavior of plates significantly, a lot of investigations on plate-foundation interaction have been conducted by researchers. Some mechanical foundation models for soil-structure interaction have been presented (Dutta and Roy 2002). Pasternak considered a shear layer which connects the springs with a shear stress interaction besides Winkler's separated springs model. Some examples for plate-foundation interaction with plate geometries different then rectangle or square are storage tanks and silos. In this scope an important part of studies are dealt with thin circular plates or plates of and annular shape. Yi-Yuan and Syracuse (Yu and Syracuse 1957) analyzed axisymmetrical bending of circular thin plates resting on Pasternak foundation analytically. To analyze bending bahavior of Reissner plates on Pasternak foundation Wang et al. (Wang et al. 1992) presented a fundamental solution and studied circular and square plates with different boundary conditions.

Among the extensive literature on plate bending and foundation interaction problem, only a few efforts were showed to investigate bending behavior of elliptical plates resting on elastic foundation. Datta (Datta 1976) used

Galerkin method to study large deflection of clamped thin elliptic plates resting on Winkler foundation and with orthotropic material properties. Zhong et al. (Zhong et al. 2005) presented a triangular differential quadrature method (TDQM) solution to analyze the flexure of uniformly loaded elliptical Reissner-Mindlin plates resting on Pasternak type elastic foundation. In this study a mixed finite element solution procedure is adopted to study bending behavior of elliptical Mindlin plates resting on Pasternak foundation. C0 type quadrilateral elements are used to construct finite element matrixes. Orthotropy of foundation with arbitrarily orientation with respect to principle directions of the plate is considered in the context of this study. Firstly a convergence and comparison study is carried out and very good agreement with literature is achieved. Parametric studies are made to investigate the effects of ellipticity, foundation parameters and orthotropy on the flexural responses of elliptical Mindlin plates interacting with Pasternak foundation.

2 Mathematical Formulation

Field equations of Mindlin plate with embedded effect of Pasternak foundation interaction are used in this study. Global coordinates (x_1, x_2, x_3) and internal forces (F, H), force couples (K, M, T) with positive directions of Mindlin plate are depicted in Fig. 1. When local coordinates (ξ, η) of orthotropic Pasternak foundation does not coincide with global coordinates orthotropy direction can be defined with an angle (θ) as seen in Fig. 1.

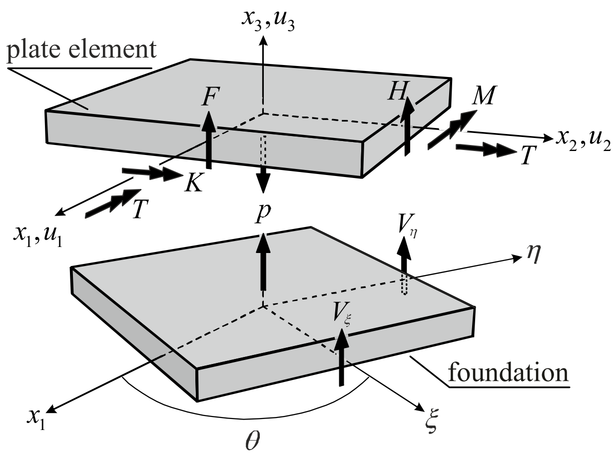

Figure 1. Mindlin plate field variables and foundation interaction

2.1 Field Equations

When coordinate transformations are made to the terms of foundation three equilibrium equations for the Mindlin plate resting on arbitrarily orthotropic foundation is obtained as;

$$\begin{aligned} &F_{,1} + H_{,2} + q - ku_3 \\ &+ G_\xi \left(\cos^2\theta u_{3,11} + 2\cos\theta \sin\theta u_{3,21} + \sin^2\theta u_{3,22}\right) \\ &+ G_\eta \left(\sin^2\theta u_{3,11} - 2\sin\theta\cos\theta u_{3,21} + \cos^2\theta u_{3,22}\right) = 0 \\ &K_{,1} + T_{,2} - F = 0 \\ &M_{,2} + T_{,1} - H = 0 \end{aligned} \tag{1}$$

where q is distributed load, k is Winkler foundation parameter, G_ξ and G_η are shear foundation parameters is ξ, η directions respectively. Constitutive equations for linear deformations of Mindlin plate are;

$$\gamma_{xz} = u_{3,1} + \Omega_1 = \frac{6F}{5Gh}$$
$$\gamma_{yz} = u_{3,2} + \Omega_2 = \frac{6H}{5Gh}$$
$$\kappa_x = \Omega_{1,1} = \frac{12}{Eh^3}\left[K - M\upsilon\right] \tag{2}$$
$$\kappa_y = \Omega_{2,2} = \frac{12}{Eh^3}\left[M - K\upsilon\right]$$
$$\kappa_{xy} = \Omega_{1,2} + \Omega_{2,1} = \frac{12T}{Gh^3}$$

where h is the thickness of the plate, E, G, υ are elasticity modulus, shear modulus and Poisson's ratio of the plate material respectively. Ω_1 and Ω_2 are rotations of cross sections with normal directions 1 and 2 respectively.

2.2 Functional

To be able to generate a mixed finite element solution depending on the Gâteaux differential, it is necessary to show that $\mathbf{Q}$ is a potential operator (positive definite and self − adjoint). $\mathbf{Q}$ must satisfy the equality $\langle \mathrm{d}\mathbf{Q}(\mathbf{y};\bar{\mathbf{y}}),\mathbf{y}^* \rangle = \langle \mathrm{d}\mathbf{Q}(\mathbf{y};\mathbf{y}^*),\bar{\mathbf{y}} \rangle$ to be a potential operator, where $\mathrm{d}\mathbf{Q}(\mathbf{y};\bar{\mathbf{y}})$ and $\mathrm{d}\mathbf{Q}(\mathbf{y};\mathbf{y}^*)$ are representatives for the Gâteaux derivatives of the operator in $\bar{\mathbf{y}}$ and $\mathbf{y}^*$ directions respectively. Defining $\mathbf{L}$ as a differential operator, $\mathbf{y}$ and $\mathbf{f}$ as vectors, field equations can be written in an operator form as $\mathbf{Q} = \mathbf{L}\mathbf{y} - \mathbf{f}$. If $\mathbf{Q}$ is a potential operator, the functional can be obtained from the field equations as $I(\mathbf{y}) = \int_0^1 \left[\mathbf{Q}(s\mathbf{y};\mathbf{y}),\mathbf{y}\right]\mathrm{d}s$ where s is a scalar quantity.

3 Parametric Studies

1

For the verification of proposed solution procedure and numerical analysis a comparison study is carried out first. The influence of ellipticity, foundation parameters and orientation of orthotropy direction in foundation is elaborated. The geometry of the plate and corresponding axis are given as in Fig. 2.

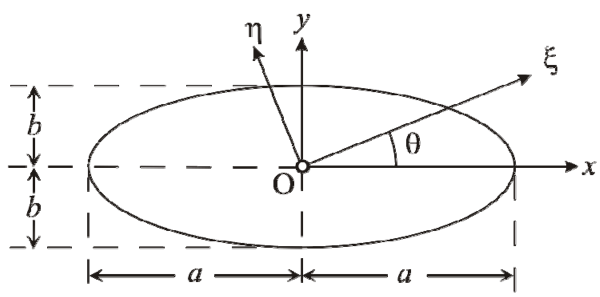

Figure 2. Elliptic plates geometry with corresponding axis

3.1 Convergence and Compare

Non-dimensional parameters are calculated as $\tilde{k} = kb^4 / D$ and $\tilde{G}_f = G_f b^2 / D$ where D is the flexural rigidity of the plate. Clamped thick circular plates ($h/b = 0.1, 0.2$) with Poisson's ratio 0.3, $\tilde{k} = 200$ and $\tilde{G}_f = 5, 20$

are studied for circular plate with *m* elements. Central deflection $\tilde{w}$ and bending moment $\tilde{K}$ are presented with a convergence study in Table 1. and an excellent agreement is achieved with the work Wang et al. (1992). It is apparent that with increasing plate thickness ratio and the shear foundation parameter a more rapid convergence is observed.

2

Table 1. Non-dimensional central deflections and bending moments of a moderately thick circular plate under uniform load *q*, $\tilde{w} = wD / qb^4 \times 1000$, $\tilde{K} = K / qb^2 \times 1000$, *m*: number of elements, $\tilde{k} = 200$.

1

			$\tilde{G}_f$= 5		$\tilde{G}_f$=20	
h / b		m	$\tilde{w}$	$\tilde{K}$	$\tilde{w}$	$\tilde{K}$
0.1	this study	48	4.52	15.07	3.40	10.58
		108	4.48	15.46	3.41	11.09
		192	4.47	15.77	3.41	11.27
		300	4.47	15.93	3.41	11.32
	Wang et al., (1991)		4.47	16.00	3.41	11.25
0.2	this study	48	4.55	1.41	3.49	1.05
		108	4.53	1.41	3.49	1.04
		192	4.53	1.42	3.49	1.04
		300	4.53	1.42	3.49	1.04
	Wang et al., (1991)		4.52	1.41	3.48	1.03

3.2 Effect of Ellipticity and Foundation Parameters

Uniformly loaded clamped plates with different aspect ratios (*a*/*b* = 1.5, 2, 3 with constant *b*) are solved. The ratio $b / h = 0.1$ and $\upsilon = 0.3$ are kept constant. Foundation parameters are $\tilde{k} = 0.32$, $\tilde{G}_\xi = 0.12$ and $\tilde{G}_\eta = 12$. Central deflection ($\tilde{w}$) and bending moment $\tilde{K}$ values are given in Table 2. It is observed that, the ellipticity is directly proportional with the deflection parameters and their maximum and minimum values are at $\theta = 0^\circ$ and $\theta = 90^\circ$, respectively. As the aspect ratio increases percent difference between the maximum and minimum values of deflection parameter also increases, e.g, for *a*/*b* = 3 it is 80.6% while for *a*/*b*=1.5 it is 29.2%. The average rate of change in non-dimensional deflection parameter becomes more significant when θ travels around 45°. For a certain value of θ, when the increase of deflection parameter related to successively increasing ellipticity is investigated, it is observed that, this difference is directly proportional with θ. For example between the aspect ratios *a*/*b* = 2 and 3 the change in deflection parameter is 4.82% at $\theta = 0^\circ$, 11.71% at $\theta = 45^\circ$ and 23.84% at $\theta = 90^\circ$.

$\tilde{K}$ is inversely proportional with the increasing aspect ratio of elliptical plate due to the decrease of the curvature of the plate along the major axis. $\tilde{K}$ is influenced considerably both from the change of θ and the aspect ratio of the elliptical plate. $\tilde{K}$ parameter takes the maximum and minimum values at $\theta = 0^\circ$ and $\theta = 90^\circ$, respectively.

Table 2. The non-dimensional central deflection and bending moment parameters of elliptical plate with constant *b* and under uniform load *q*, $\tilde{w} = w / Dqb^4 \times 1000$, $\tilde{K} = K / qb^2 \times 1000$

	$\tilde{w}$					$\tilde{K}$				
a/*b* θ	0°	30°	45°	60°	90°	0°	30°	45°	60°	90°
1,5	16,39	17,38	18,48	19,73	21,18	42,38	44,98	47,90	51,10	54,90
2	18,05	19,78	21,86	24,39	27,59	33,00	36,85	41,40	46,93	54,05
3	18,92	21,33	24,42	28,49	34,17	25,20	29,28	34,58	41,68	51,98

Non-dimensional bending moment $\tilde{M}$ parameters are tabulated in Table 3. With the increasing aspect ratio of the elliptical plate, since the curvature along the minor axes increases, the non dimensional bending moment parameter $\tilde{M}$ also increases. The change of θ has a considerable influence on the $\tilde{M}$ parameter. In Table 3. non-dimensional torsion moment ($\tilde{T}$) parameters at the center of elliptic plate has been presented. In the cases of $\theta = 0^\circ$ and $\theta = 90^\circ$, $\tilde{T}$ parameter is 0 due to the symmetry. Deviation of the orthotropy direction from principal axis of the plate removes the flexure symmetry of plate and torsional moments appear at the symmetry center. However, the influence of the orthotropy direction on $\tilde{T}$ parameter is less compared to the bending moment parameters.

Table 3. The non-dimensional bending and torsional moment at the center of elliptical plate with constant *b* and under uniform load *q*, $\tilde{M} = M / qb^2 \times 1000$, $\tilde{T} = T / qb^2 \times 1000$

	$\tilde{M}$					$\tilde{T}$				
a/b θ	0°	30°	45°	60°	90°	0°	30°	45°	60°	90°
1,5	62,43	67,80	73,90	80,88	89,35	0,00	1,43	1,81	1,79	0,00
2	64,38	72,60	82,65	95,13	111,73	0,00	1,00	1,40	1,53	0,00
3	64,40	74,85	88,55	107,05	134,38	0,00	0,47	0,75	0,95	0,00

Conclusion

In this study bending behavior of moderately thick elliptic plates resting on arbitrarily orthotropic Pasternak foundations is investigated. Solution procedure is based on a mixed finite element formulation. Mindlin plates field equations are used with linear deformations. Gâteaux differential is used to obtain the mixed finite element formulation. Four nodded isoparametric quadrilateral elements are used over plate domain. The nodal unknowns are; one lateral translation, two rotations, two transverse shear forces, two bending and one twisting moments. Presented solution method is verified with comparison study successfully. Parametric investigation of the clamped elliptical plate is dealt with orthotropy orientation of foundations, foundation parameters and ellipticity parameters. Deflection and moment responses are presented in tables. It is exposed that foundation parameters, principle directions of the orthotropy, and ellipticity have significant effects on the bending responses of elliptic plates.

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