**Elastic Curve**


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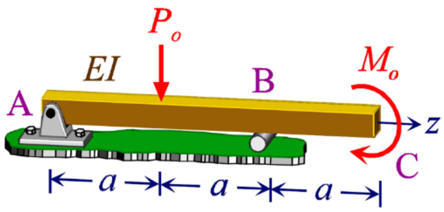
**Question :** An overhanging beam is loaded as shown in the Figure. Determine the ratio $P_o\,a\,/\,M_o$, which will not cause any vertical deflection at the free end of the beam using both the Mohr's method and the clamped beam method.

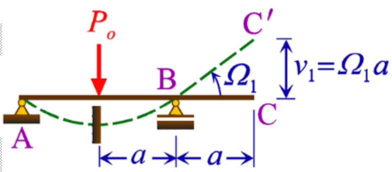



**Solution :**

The problem will be solved by superposing the results of the singular load $P_o$ and the bending moment $M_o$.

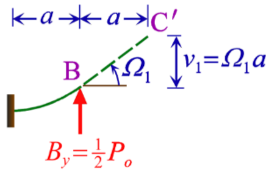
- The clamped beam method:

The support reactions are obtained using the equilibrium equations for the singular load,




$$A_y = B_y = \tfrac{1}{2}P_o \uparrow$$

The rotation and the deflection at the end of the beam are obtained as,

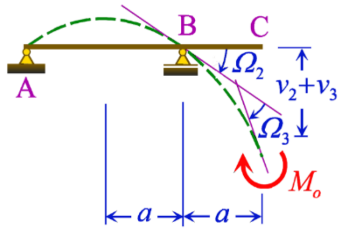



$$\Omega_1 = -\frac{B_y L^2}{2EI} = -\frac{\left(\frac{1}{2}P_o\right)a^2}{2EI} = -\frac{P_o a^2}{4EI}$$

$$v_1 = \Omega_1 a = -\frac{P_o a^3}{4EI}$$

- The Mohr's method:

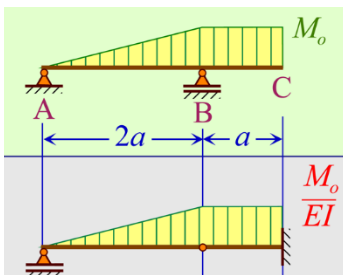
The support reactions are obtained using the equilibrium equations for the singular moment,




$$A_y = \frac{M_o}{2a}\downarrow \quad ; \quad B_y = \frac{M_o}{2a}\uparrow$$

The curvature of the actual beam is applied as a fictive load on the conjugate beam. The moment equilibrium is used for segment AB and the reaction at the point $B_y$ is obtained,

$$\sum M_A = 0;\quad (2a)\bar{B}_y - \left(\frac{2}{3}(2a)\right)\left[\frac{1}{2}(2a)\left(\frac{M_o}{EI}\right)\right] = 0 \quad\Rightarrow\quad \bar{B}_y = \frac{2M_o a}{3EI}$$



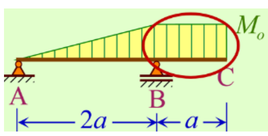

The rotation is determined based on the analogy $\Omega \Leftrightarrow \bar{T}$,

$$\Omega_2 = \frac{2M_o a}{3EI}$$

The deflection $v_2$ due to the rotation $\Omega_2$,

$$v_2 = \Omega_2 a = \frac{2M_o a^2}{3EI}$$

The deflection $v_3$ due to the moment $M_o$ (The segment BC),



$$v_3 = \frac{M_o a^2}{2EI}$$

The total deflection is obtained by superposing the results and it must be equal to zero in order to validate "no deflection" condition,

$$v = v_1 + v_2 + v_3 = -\frac{P_o a^3}{4EI} + \frac{M_o a^2}{2EI} + \frac{2M_o a^2}{3EI} = 0$$

The ratio is determined using the above equation,

$$\frac{P_o a}{M_o} = \frac{14}{3}$$